Research and Practice in Computer-Aided Structural Engineering

by
William McGuire

The Twenty-Third Arthur J. Boase Lecture in Civil Engineering

March 22, 1988

Department of Civil, Environmental and Architectural Engineering
College of Engineering and Applied Science
University of Colorado at Boulder
Boulder, Colorado 80309-0428
RESEARCH AND PRACTICE IN
COMPUTER-AIDED
STRUCTURAL ENGINEERING

By

William McGuire
Professor of Structural Engineering
Cornell University

Arthur J. Boase Lecture
Department of Civil, Environmental and
Architectural Engineering
University of Colorado at Boulder

March 22, 1988
ARTHUR J. BOASE  
1892-1949

Arthur J. Boase was a native Coloradan, born, brought up, and educated in his home state. After receiving his B.S.C.E. degree from the University of Colorado at Boulder in 1918, he worked as a practicing engineer in the Boulder-Denver area for 11 years. He then moved East to teach and become head of the Civil Engineering Department at Pennsylvania Military College. Subsequently, he became Regional Engineer for the Portland Cement Association in Philadelphia before moving to Chicago as Manager of the Structural Bureau for PCA in 1932.

Art Boase was held in high esteem by professionals in the field of structures and construction with particular emphasis on reinforced concrete. Because he recognized the need for better theories and design tools, he used his imagination and enthusiasm to promote new ideas and innovative research. The results of much of his research have been published in PCA Bulletins. Among his many other contributions, he was a significant participant in the establishment of the Reinforced Concrete Research Council, the ACI Reinforced Concrete Design Handbook, the Joint Committee on Standard Concrete Specifications, and the ACI Building Code Committee.

Art Boase was a loyal alumnus. In his will he provided that upon the death of his widow, his estate would be left to the Department of Civil Engineering at the University of Colorado. In appreciation of his generosity and thoughtfulness, the Arthur J. Boase Lecture Series was established in 1971 in his memory.

WILLIAM McGUIRE

Professor William McGuire has been on the faculty of Cornell University since 1949. He has been instrumental in introducing computer methods to structural engineering practice. He is the author of "Steel Structures" (1968) and co-author of "Matrix Structural Analysis" (1979).

RESEARCH AND PRACTICE IN COMPUTER-AIDED STRUCTURAL ENGINEERING

by

William McGuire

Professor of Structural Engineering  
Cornell University

Arthur J. Boase Lecture  
University of Colorado at Boulder  
March 22, 1988

ABSTRACT

The paper is a commentary on one part of computer-aided engineering: the analysis and design of civil engineering structures, particularly steel structures. Twelve years of research on the application of interactive computer graphics to the subject are summarised. This is followed by impressions of present computer-aided design methods, the state of affairs in hardware and software, and the impact of computer workstations on practice. Some possible directions for research and practice are then suggested. Emphasis is on practice and, in particular, on ways in which the computerised treatment of frequently encountered structural stability problems may be improved. Comments on problems of technology transfer in civil engineering are also presented.
INTRODUCTION

The computer era began over forty years ago but the author was attracted to computers rather late, when graphics became a reality in the mid 1970's. At that time a Cornell Professor of Architecture, D. P. Greenberg, assembled the nucleus of a computer graphics laboratory - an Evans and Sutherland picture system controlled interactively by a digitizing tablet and served by a PDP-11 minicomputer. It seemed obvious that here at last was a computerised medium that could let structural engineers function in the way they should function: in direct control of their work, in visual contact with the results of analysis and design trials, and with a computer small in size but powerful enough to enable them to utilise any analytical tool appropriate to a given task. To fund research towards this end, the author and his colleagues, R. H. Gallagher and J. F. Abel, sought and obtained the support of the National Science Foundation.

From the beginning, Cornell research on the application of interactive computer graphics to the practice of structural engineering had certain objectives and certain restrictions. Analysis was viewed as an integral part of design, which means that the engineer should have the ability to call immediately upon either analysis routines or design sequences. He should be able to restart, redo, or enter any place in the process in almost any order (Reference 1). It was decided to emphasize problems involving nonlinearity, three-dimensionality, and dynamic as well as static behavior. It was felt that these criteria would provide proper research for structural engineering graduate students and define a field distinct from the domain of commercial software developers. Incursion into the drafting and detailing parts of design was avoided. Although these are at least as important as anything we might do, they were not our business and they were being addressed energetically by software firms.

Our aim was - and continues to be - to contribute to practice. But we believe we can do it best through university research which, we hope, also advances the understanding of structural behavior.

The Cornell work has been focused on steel structures, but this was more by accident of the author’s interest in that field than by rational selection. To correct that shortcoming, other Cornell researchers have lately started to extend the work to concrete and mixed construction.

Figure 1, from Reference 2, shows some of the products of the early research, four displays of the results of a geometric and material nonlinear analysis of a planar frame: the deflected structure, a response curve, a yield surface and force point track, and a color picture of force distribution and plastic hinges. It
is primitive, but eleven years ago it was the state of the art of the application of interactive computer graphics to common structures.

By 1980, an interactive graphics preprocessor capable of fully defining three dimensional structures and their loads had been developed (Figure 2, from Reference 3). This was linked to an integrated analysis/design program for statically loaded structures by 1984 (Figure 3, from Reference 4). Still in this chain, an earthquake analysis and design program was produced in 1986 (Figure 4, from Reference 5). Four years ago it was decided to branch out and investigate the capabilities of emerging supercomputers by macromodelling large steel structures and micromodelling individual members and small subassemblages. Figure 5, a result of a full nonlinear time-history analysis of a three dimensional frame reported in Reference 6, is an example of the former and Figure 6, showing local buckling of a beam under a moment gradient, illustrates the latter (Reference 7). Both are from studies reported in early 1988.

Cornell research in both the practice oriented direction and the study of fundamental analytical and behavioral problems continues. Most of the more basic research involves extension of the macro- and micromodelling just illustrated. This means studies in the use of very powerful computers through parallel processing, large scale nonlinear dynamic analysis of earthquake resistant structures, detailed analytical studies of local and overall stability phenomena, and linkage of analysis and experiment. On the practice side, the research is attempting to advance the cause of computer aided engineering by extending and applying the software already developed to amenable but still inadequately treated problems.

Figure 1: Plane Frame Analysis, 1977
(c) Yield Surface

(d) Force Distribution

Plane Frame Analysis, 1977, Cont'd.

Figure 2: Frame Preprocessor, 1980

Figure 3: Static Analysis, 1983
PRESENT PRACTICE

Although the author's direct contact with practice is limited, a few things seem clear.

One is that just about every structural engineer is using a computer and every engineering student now preparing for practice is getting instruction in programming and the use of computers. Thus at least a minimal level of computer literacy and computer use have been established and are no longer something in the future.

With respect to hardware, the full spectrum is in use, from PC's to the mini supercomputers now becoming available. But the author's guess is that most practicing structural engineers are operating with hardware nearer the lower end of the line, with personal computers or in-house minicomputers such as the VAX 11-750. Bigger computers are still confined to larger engineering or architectural-engineering organisations or to service bureaus that have the resources for large scale or complex analysis. A key point, to be addressed momentarily, is the probable impact on the hardware environment of the workstations that are rapidly becoming available.

Regarding software, a great variety of programs are in use. Perhaps most of these are in-house programs written by design engineers for local use. But many packaged programs are on the market as well.

The impression is that most of the reliable programs in circulation employ linear elastic analysis, some design features, and some graphics. Most are intended for PC use and a number are menu-driven and employ a mouse for user interaction. A recent example is Fujitsu America's software packages ELM 2-D and ELM 3-D for interactive stress analysis on IBM PC XT/AT or compatible equipment. Intergraph's CADD system is an example of a move to integrate the full structural analysis-design-drawing process.

To the author's knowledge, except for elementary first order plastic analysis or approximate second order elastic analysis of the P-G variety, none of the inexpensive commercial programs that are commonly available contain provisions for nonlinear analysis.

Thus some of the things that were legitimate university research ten years ago have been embraced by practice, some seem imminent in their applicability and require only minor further develop, and some have led to broadening or deepening fields of advanced research.
HARDWARE AND SOFTWARE

At comparable ages, earlier technological developments - steam power, radio, aircraft, etc. - had attained states of relatively steady, predictable growth. The mistaken, the misdirected, and the unwisely speculative had largely been driven from the scene. Not so in computers. On the contrary, these are most turbulent times. The complexity of the medium and its potential are unprecedented. It continues to spawn new ideas and new developments, and the end is not in sight. Thus this is an exciting period but it is also a difficult and dangerous one.

On March 4 of this year the New York Times carried an article on setbacks for artificial intelligence and the companies that thought they would prosper by providing the technology to make computers think. Of eleven major companies mentioned, nine have experienced large losses, major layoffs, and, in one case, bankruptcy. Two days later the same paper reported the introduction of Digital's VAX 8800 series computer, a competitor for IBM in the mainframe field. But what might be expected to be a day of celebration for DEC was greeted by a four dollar drop in its shares because, "the analysts said that Digital was expected to announce new software with the new computers, but when the company did not, they lowered their earnings projections. We were looking for the queen to come in with her court, and we only got the queen, said one".

There are ongoing attempts to stabilise portions of the field through unification and the development of standards. But in the main, these are still in an unsatisfactory state. There remain different operating systems - UNIX, VMS, MS-DOS, etc. - and each system has its variants. The same is true of graphics standards. PHIGS is a nominal standard, but it is still not a mature, established one. At the programming level there are Basic, Fortran, C, etc. - and each of their variants.

We at Cornell have been caught in this morass in the process of converting some of the programs mentioned earlier from the advanced but unique system on which they were developed to a workstation environment. When we started a year ago we viewed the job as analogous to moving a trainload of sand from a broad gage track to a standard gage one by hoisting each car off its wide tracks and installing it on standard ones - a tedious, heavy lifting, but elementary process. Now I wish it were that simple. It has turned out to be more like moving the sand to new cars by grain or, in our case, bit by bit.

If the turbulence results in progress and order emerges from chaos, the frustrations of the present will prove worth enduring. But, in the process, through-
65 dedicated servers. A typical Athena workstation has a 1 mips processor, a megapixel (1000x1000) display, and three or four megabytes of memory. It also has a mouse, a 30-70 megabyte disk, and an Ethernet network interface. Actual hardware in use includes DEC VAX Station II and VAX Station 2000 systems, and IBM RT PC desktop models (Reference 8). What goes on in research universities is not often what practitioners find affordable or useful. But in this case, the trends of increasing hardware power, decreasing hardware costs, accelerating software development, and increasingly sophisticated computer application courses in universities - ill controlled as each trend may be - should combine to make hardware of this sort affordable to most practitioners within the next five years.

Figure 6: Micromodel, 1987

Figure 7: MicroVAX Workstation, 1988
A DIRECTION FOR PRACTICE - ONE AREA

In spite of present problems, the future for computer-aided structural engineering remains limitless. As an illustration of possibilities, ways in which research of the type conducted at Cornell can be used to advance the practical treatment of problems in one area - the stability of steel structures - will be described.

It is the nature of steel structures that all of their strength limit states - except for fatigue, fracture, and tension member yielding - are in fact stability limits. Even the exceptions can be included on the basis that they are limits of resistance only when they result in major kinematic instability, i.e., large rigid body motion of part or all of a structure. For this reason the AISC LRFD Specification (Reference 9) is a good guide to needs in stability design since, of necessity, it is replete with relevant requirements, rules, and guidelines: formulas such as the column equation, tables such as Table B5.1 which guards against premature local buckling by placing limits on width-to-thickness ratios, and member dimensioning rules such as those intended to prevent dishing instability in pin-connected members. The intention of the LRFD Specification is, "to provide design criteria for routine use and not to cover infrequently encountered problems which occur in the full range of structural design". Computerized design aids that adhere to this guideline should be the ones most used by the profession.

LRFD SPECIFICATION CONFORMANCE CHECKING - The first computer-based tool to be mentioned is one not related to Cornell research but is, rather, an electronic version of the LRFD Specification being developed by Visual Edge Software, Ltd. under the direction of Dr. M. H. Ackroyd, with S. J. Fennes and the author as consultants. ELRFD, as it is called, is software that will run on a PC. It will enable automated and semi-automated interpretation of the LRFD Specification and evaluation of structural components for conformance with the LRFD provisions. The aim is the complete classification of the limit states of steel building structures and the automated evaluation of all of the design equations and dimension provisions in the Code. Figure 13 is a diagram of the modules of the system. The Input Editor, Element Type Processor, and Limit States Processor organise the input. The Object Processor organises the links between modules, and the Decision Table Processor controls the evaluation of requirements and the return of a clear statement of the member's compliance or noncompliance with the Code. Figure 14 contains displays of the top level decision table for the conformance check of strong axis flexural buckling and the tables for computing $\phi_x$, $P_{cr}$, and $P_n$, values required in making this evaluation.

If our aims are realised, ELRFD will include all parts of the Specification that can be reduced to logical statements. It will be an elegant way to invoke and evaluate all of its specific rules, requirements, and formulas and to minimise miscalculation. It is also central to the treatment of many stability considerations: operations such as the use of Table B5.1, the numerical evaluation of Equation H1-1, and the parsing of Table A-F1.1 on the various stability limits on beams. Although it has not been attempted yet, software such as ELRFD could be incorporated in a program such as CU-STAND, with analysis results from the latter sent to ELRFD for evaluation, and interpretation of the outcome of the code checks facilitated by STAND's graphic displays. Alternatively, equations to be evaluated could be compiled interactively by using the page shown in Figure 12, with evaluation logic and results incorporated directly in STAND.

But there is more to the treatment of stability problems than this. There are many aspects of basic stability design that go beyond checking for conformance with standardized rules. In the author's opinion, treatment of such stability problems by other than the simplified procedures that are already available will need more than a standard personal computer. In present hardware terms, minimum requirements are the computing power of a VAX 2000 with networking support, interactive graphics, and the ability to run programs both interactively and in batch mode.

Turning to programs that can handle the more or less routine stability problems that fall within the direct coverage of a code such as AISC LRFD and perhaps also enable the engineer to deal independently with other frequently encountered but more complex conditions, there are a number of possibilities. The suggestions to be made follow more or less the order in which the needs they address appear in the LRFD Specification.

TYPES OF CONSTRUCTION - Two basic types of construction are defined in the standard. Analysis programs should be capable of calculating member forces for both Fully Restrained (FR) and Partially Restrained (PR) construction. Consideration of rotation in connections and the effect of connection flexibility on the stability of the frame should be possible. The preprocessor should include provisions for the interactive definition of partially rigid connections (cubic spline, multi-linear, or other types of moment rotation curves such as the Ramberg-Osgood curve in Figure 15). Definition of "leaning" columns through the use of fully flexible connections should also be possible. The aim should be to enable full application of the provisions of LRFD Section A2.2, Types of Construction. Scope would be limited only by the extent and reliabil-
ity of connection behavior data.

DESIGN BASIS - Another basic provision is the permission to use either elastic and plastic (inelastic) analysis contained in Section A5.1, Required Strength at Factored Loads. This, coupled with later requirements for considering second order effects (e.g., LRFD Sections B4, Stability and C1, Second Order Effects) means that four basic methods of analysis are needed:

1) first order elastic (linear elastic)
2) second order elastic (geometric nonlinear)
3) first order inelastic (material nonlinear)
4) second order inelastic (full nonlinear)

Figure 16 is a sample of the results obtainable from a program of each type incorporated in CU-STAND. The roles of these programs in stability checking will be mentioned below.

STABILITY - The basic statement on stability per se is the precept in Section B4: “General stability shall be provided for the structure as a whole and for each compression element”. This can be met by a battery of programs or tools of different levels of complexity and rigor. Chief among them are the following, some of which exist in practically useful form, some as research supporting programs, and some in only rudimentary, but improvable, form:

i) Aids for formula checking- These are routines that could supply data needed by ELRFD or a similar system for the use of the Specification’s stability checking equations. These needs include: (a) routines for calculating elastic and inelastic effective length factors from eigenvalue analyses of complete frames or subassemblies, and (b) a routine to calculate the moment magnification factor B1 or the subsidiary factor Cm for transversely loaded members by “rational analysis”. Figure 17 shows effective lengths determined by an eigenvalue analysis procedure that incorporates an inelastic modulus reduction factor, r. Figure 18 displays B1 values obtained as the ratio of results of second and first-order elastic analyses. An obvious preferred alternative is the direct use of the second order analysis results in the Code formulas.
ii) Simple second order analysis programs- These are programs of the types mentioned above that use simple discrete elements and concentrated plasticity modeling, and that are capable of detecting limit points in large systems. Figure 19 illustrates the calculation of the stability limit of a frame in which the limit of resistance is a function of both geometrical and material nonlinearities.

iii) Critical load analysis programs- These are eigenvalue routines of the type already mentioned that are capable of calculating elastic and inelastic critical loads of large systems. Figure 20 shows the elastic critical load and out-of-plane buckling mode of a Vierendeel truss calculated by the program in CU-STAND.

iv) Refined second order analysis programs- These are fiber element or finite element programs that include provisions for modelling spreading plasticity, residual stresses, and initial imperfections, and that are capable of determining limits of flexural stability in isolated members and small subassemblages. Figure 21 shows the response of a small portal frame obtained from a program of this type compared with the response calculated by the second order inelastic program in CU-STAND. STAND's simpler second order program, while reliable in predicting the response of larger systems in which the influence of residual stresses and local imperfections becomes relatively unimportant, is incapable of showing the more significant influence of these effects on small systems.

The aim in having this battery of four facilities of different levels is to provide almost all of the analytical tools an engineer might want to draw upon to satisfy the stability provisions of Sections B4 (Stability), C1 (Second Order Effects), C2 (Frame Stability), E1 (Effective Length and Slenderness Limitations), E2 (Design Compressive Strength), and H1.2 (Doubly and Singly Symmetric Members in Flexure and Compression). Which he would use in a given case would depend upon the complexity or size of the problem, his expertise, his taste, and the design economics. Omitted from this list of programs are direct analysis of torsional-flexural instability conditions and modelling of shear-resistant infilling. These basic needs will be discussed later.

COMBINED FLEXURE AND COMPRESSION - Perhaps the most commonly used routine check for stability is the interaction equation of Section H1.2 of the Specification. Indispensable to its application is input that can be used by ELRFD or a similar evaluation tool. This can be supplied either by linear elastic analysis coupled with k factor and Cm determination or directly by second order elastic analysis, as mentioned earlier. The option of using inelastic analysis requires further discussion, which will also be offered later.

ADDITIONAL NEEDS - The tools described above can all be founded on
rational mechanics and universally accepted concepts of analysis and behavior. This is important in gaining acceptance of the programs and ensuring their relative durability. But to provide a complete set of computerized tools to meet the demands of routine design some additional features are needed, and the present state of the art of structures is such that these must include procedures that are more ad-hoc in nature. There are two particular areas:

i) The effects of torsion - Common cases of torsion and the torsional-flexural stability of individual beams and beam columns are treated adequately by the formulas in the LRFD Specification and thus by computerized tools such as ELRFD. But the analysis of nonuniform torsion and the effect of coupled torsion and flexure on the elastic and inelastic stability of systems are not covered. There is a large literature on these subjects and there are many programs that can be of use. Figure 22 (Reference 11), for example, illustrates the results of a second order elastic analysis of a beam column in which both warping and St. Venant resistance are included and a warping spring concept is used to account for the bimoment effect at the supports. But more is needed to approximate the full effect of out-of-plane member behavior on the stability of frames, particularly in the inelastic range. And much of what is needed must be essentially empirical for the time being.

ii) Element modelling - Procedures that should be provided as addi-tions or options to the basic analysis routines are programs for modelling finite joint size, panel some effects, member shearing deformations, and the characteristics of shear-resistant infills.

INELASTIC ANALYSIS - A last essential that requires consideration is the use of inelastic analysis. It is permitted in LRFD and can be used for the design of continuous beams in the ways that have become standard practice. But there are two questions regarding the way it has been included in the Specification that have to be addressed before it can be used routinely and efficiently where frame stability may limit resistance. The first concerns the absence of clear guidelines for the consideration of second order effects in inelastic analysis comparable to the guidelines for elastic analysis. The second is whether adequate provision is made for the redistribution of moments that result from axial forces significant enough to influence member yielding, one of the principal ways in which nonlinear analysis simulates system behavior. Research to supply the desired guidelines and reconcile the apparent discrepancies between the code approach and the basic theory is underway. When this research has reached the point of producing reliable, office usable, computerized inelastic analysis/design procedures, they should be added to the basic tools to complete the set.

DESIRABLE ITEMS - In the body of the LRFD Specification and in its appendices there are references to other stability problems that are encountered often enough to warrant computerised design aids, even though they may not be of every-day concern. Among them are the following:

i) Web-Tapered Members- These could be treated by a program based on Appendix F4.

ii) Plate Girders- Stability problems associated with tension field behavior that are codified in Appendix G3 are covered in ELRFD. The addition of an alternate tension field design method selected from a study of the latest developments in that subject is desirable however.

iii) Flexural-Torsional Buckling- In addition to the coverage considered essential and described earlier, there would be use for programs that go beyond the torsional-flexural buckling equations that appear in the body of the Specification or Appendices E and F (and are therefore included in ELRFD). An example is a program that would cover numerous conditions of elastic lateral-torsional buckling somewhat after the fashion of a paper by Nethercot and Rockey (Reference 10). Also desirable is a program that would consider stability limits that are covered in the AISI Cold Formed Steel Specification and are relevant to hot rolled steel, but are not presently in LRFD.
Figure 15: Moment-rotation Curves

Figure 16: Analysis Results

Figure 17: Inelastic Effective Lengths

Figure 18: B1 Factor
Figure 19: Inelastic Frame Stability

Figure 20: Elastic Critical Load

Figure 21: Portal Frame Analysis
SUMMARY

In this review the author has tried to do several things: First, to emphasize common stability problems by stressing the stability conditions covered in the AISC LRFD Specification. Second, to consider programs that can solve practical stability problems on affordable hardware. And third, to refer to tools and programs that already exist in rudimentary or prototype form or that can be developed within the next few years without requiring significant research. Within these guidelines he has tried to make a case for nonlinear as well as linear analysis, three- as well as two- and one-dimensional analysis, and tools based as closely as possible on fundamental mechanics and structural behavior concepts. The importance of keeping the user in control of any program and informed of the results through the medium of interactive computer graphics has also been stressed.

Cornell programs have been used to illustrate most of the suggestions. As originally developed these programs had several shortcomings, vestiges of which remain: 1) They were developed on advanced, specialized hardware and were not transportable. 2) They were developed by graduate students, which meant that the coding was spotty and certainly not of commercial grade. 3) Although dealing with practical concerns, they were developed as components of research in structural engineering. We made some points through our attempts to show ways in which interactive graphics can be used advantageously in design and in education. But students were never required to develop comprehensive, office-usable, design routines. We always stopped short of what was at the time commercial software development.

We are now well along in the transportation of our programs to a work-station environment. But it is a slow, frustrating, process. Establishment of the programs on workstations such as the DEC Micro VAX and Sun series will reduce or eliminate most of their shortcomings. They will be transportable and more modular and thus can be modified more easily, but they will still not be of commercial grade. And, since our functions are teaching and research, there will be limits on problem size: the programs can handle real structures of moderate size - say 20 story buildings - but not extremely large systems. But even with these limitations, the programs illustrated - and many of the other, comparable programs that are available elsewhere - can be useful in the development of office-usable computerised aids.

The point is that many of the things that can be done to advance computer aided structural engineering do not require research; the methods exist. It is true, however, that realisation of office-usable aids requires a large amount of
effort - most of it in software development in the no-man's land between research and commerce. It involves programming that goes beyond the needs of graduate theses but that does not promise a clear return for commercial software developers. An example of a similar situation that has received proper attention is the development of efficient eigenvalue routines, programs that solve the equation:

$$(A - \lambda B)\mathbf{x} = 0$$

This problem wasn't dealt with adequately until the Argonne National Laboratory under the National Activity to Test Software Project (NATS), in collaboration with teams from several universities and government laboratories, organised the development of EISPACK, a widely used eigenvalue package consisting of a systematised collection of Fortran subroutines. If millions of dollars could have been spent on one equation, the structural engineering profession should be interested in a comparable effort to deal with its common stability problems in a computerised way. But, given the conditions of American practice, organising, directing, and financing the enterprise would have to be a cooperative venture. Perhaps the Structural Stability Research Council could be the catalyst and provide the intellectual drive, the ASCE and AISC the moral and organisational support, and the NSF much of the funding. If such an effort is not undertaken, progress will continue to be made, but it will be disorganised - practical application of many of the techniques that are ready for use will come about slowly and haphazardly.

ACKNOWLEDGEMENTS

The author appreciates the invitation from the Department of Civil Engineering to give this twenty-third Arthur J. Boase Lecture in Civil Engineering. He also wishes to acknowledge the contributions of his colleagues J. F. Abel, R. H. Gallagher, and C. H. Cooley, and the financial support of the National Science Foundation. Above all, he owes a profound debt of gratitude to many graduate students: first to R. D. Ziemian who prepared the examples in this paper and, next, to those who went before and developed the programs mentioned: J. L. Gross, T. A. Mutryn, C. I. Pesquera, M. Gattass, J. G. Orbison, Y. B. Yang, S. I. Hilmy, K. N. Loo, J. L. Castañer, S. N. Sutharshana, D. W. White, and J. F. Hajjar.
References


ARThUR J. BOASE LECTURE SERIES 
IN CIVIL ENGINEERING

3. Realizing the Potential of Materials by Glenn Murphy, April 1975.
10. Shaking Table Experiment in Earthquake Engineering by Ray Clough, September 1981.
15. Finite Element Modeling of Concrete Structures by Berger, Cervenka and Ottesen, September 1983.